Skew-symmetric numerical schemes for stochastic differential equations

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I will explain a new simple and explicit numerical scheme for time-homogeneous stochastic differential equations. The scheme is based on sampling increments at each time step from a skew-symmetric probability distribution, with the level of skewness determined by the drift and volatility of the underlying process. I will discuss weak and strong convergence to the underlying diffusion as the step-size is reduced. I will then discuss the problem of simulating from the limiting distribution of an ergodic diffusion process using the numerical scheme with a fixed step-size. The problem is of fundamental interest in Bayesian statistics, where Monte Carlo sampling algorithms based on simulating from the equilibrium distribution of a Markov process play a crucial role. I will show conditions under which the numerical scheme converges to equilibrium at a geometric rate, and quantify the bias between the equilibrium distributions of the scheme and of the true diffusion process (which, in the statistical context, is the posterior distribution of interest). Notably, our results do not require a global Lipschitz assumption on the drift, in contrast to those required for the Euler–Maruyama scheme for long-time simulation at fixed step-sizes (otherwise known as the unadjusted Langevin algorithm, or ULA).

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